

PROBLEM SET - 3 (PROBABILITY DEFINITIONS, JOINT AND CONDITIONAL PROBABILITY)

ECO 104 (Section 8)
Instructor: Shaikh Tanvir Hossain

Due Date : 25th November, 2023, Saturday (submit in class), individual assignment

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You should write the solutions neatly in paper and submit in class. **This is NOT a group assignment, this is an individual assignment**, so please do the problems and submit individually! Please ask me in Ed if you have any question.

§. Theory and Methods

1. From Anderson et al. (2020), Chapter 4, #30

Suppose that we have two events, A and B , with $P(A) = .50$, $P(B) = .60$, and $P(A \cap B) = .40$

- Find $P(A | B)$.
- Find $P(B | A)$.
- Are A and B independent? Why or why not?

2. From Anderson et al. (2020), Chapter 4, #31

Assume that we have two events, A and B , that are mutually exclusive. Assume further that we know $P(A) = .30$ and $P(B) = .40$.

- What is $P(A \cap B)$?
- What is $P(A | B)$?
- A student in statistics argues that the concepts of mutually exclusive events and independent events are really the same, and that if events are mutually exclusive they must be independent. Do you agree with this statement? Use the probability information in this problem to justify your answer.
- What general conclusion would you make about mutually exclusive and independent events given the results of this problem?

3. From Anderson et al. (2020), Chapter 4, # 39.

The prior probabilities for events A_1 and A_2 are $P(A_1) = .40$ and $P(A_2) = .60$. It is also known that $P(A_1 \cap A_2) = 0$. Suppose $P(B | A_1) = .20$ and $P(B | A_2) = .05$.

- Are A_1 and A_2 mutually exclusive? Explain.
- Compute $P(A_1 \cap B)$ and $P(A_2 \cap B)$.
- Compute $P(B)$.
- Apply Bayes' theorem to compute $P(A_1 | B)$ and $P(A_2 | B)$.

4. From Anderson et al. (2020), Chapter 4, #40.

The prior probabilities for events A_1, A_2 , and A_3 are $P(A_1) = .20$, $P(A_2) = .50$, and $P(A_3) = .30$. The conditional probabilities of event B given A_1, A_2 , and A_3 are $P(B | A_1) = .50$, $P(B | A_2) = .40$, and $P(B | A_3) = .30$.

- Compute $P(B \cap A_1)$, $P(B \cap A_2)$, and $P(B \cap A_3)$.
- Apply Bayes' theorem, equation, to compute the posterior probability $P(A_2 | B)$.
- Use the tabular approach to applying Bayes' theorem to compute $P(A_1 | B)$, $P(A_2 | B)$, and $P(A_3 | B)$.

§. Applied Problems

5. From Anderson et al. (2020), Chapter 4, #25

A 2018 Pew Research Center survey (Pew Research website) examined the use of social media platforms in the United States. The survey found that there is a .68 probability that a randomly selected American will use Facebook and a .25 probability that a randomly selected American will use LinkedIn. In addition, there is a .22 probability that a randomly selected American will use both Facebook and LinkedIn.

- What is the probability that a randomly selected American will use Facebook or LinkedIn?
- What is the probability that a randomly selected American will not use either social media platform?

6. From Anderson et al. (2020), Chapter 4, #27.

A marketing firm would like to test-market the name of a new energy drink targeted at 18- to 29-year-olds via social media. A study by the Pew Research Center found that 35% of U.S. adults (18 and older) do not use social media (Pew Research Center website, October 2015). The percentage of U.S. young adults age 30 and older is 78%. Suppose that the percentage of the U.S. adult population that is either age 18-29 or uses social media is 67.2%.

- What is the probability that a randomly selected U.S. adult uses social media?
- What is the probability that a randomly selected U.S. adult is aged 18-29?
- What is the probability that a randomly selected U.S. adult is 18-29 and a user of social media?

7. From Anderson et al. (2020), Chapter 4, #29.

High school seniors with strong academic records apply to the nation's most selective colleges in greater numbers each year. Because the number of slots remains relatively stable, some colleges reject more early applicants. Suppose that for a recent admissions class, an Ivy League college received 2851 applications for early admission. Of this group, it admitted 1033 students early, rejected 854 outright, and deferred 964 to the regular admission pool for further consideration. In the past, this school has admitted 18% of the deferred early admission applicants during the regular admission process. Counting the students admitted early and the students admitted during the regular admission process, the total class size was 2375 .

- Let E be the event that a student who applies for early admission is admitted early
- R be the event that a student who applies for early admission is rejected outright
- and D be the event that a student who applies for early admission is deferred to the regular admissions pool.

- Use the data to estimate $P(E)$, $P(R)$, and $P(D)$.
- Are events E and D mutually exclusive? Find $P(E \cap D)$.
- For the 2375 students who were admitted, what is the probability that a randomly selected student was accepted during early admission?
- Suppose a student applies for early admission. What is the probability that the student will be admitted for early admission or be deferred and later admitted during the regular admission process?

8. From Anderson et al. (2020), Chapter 4, #32.

Consider the following example survey results of 18 - to 34 -year olds in the United States, in response to the question "Are you currently living with your family?"

	Yes	No	Totals
Men	106	141	247
Women	92	161	253
Totals	198	302	500

- Develop the joint probability table for these data and use it to answer the following questions.
- What are the marginal probabilities?

- (c) What is the probability of living with family given you are an 18-to 34-year-old man in the United States?
- (d) What is the probability of living with family given you are an 18-to 34-year-old woman in the United States?
- (e) What is the probability of an 18-to 34-year-old in the United States living with family?
- (f) If, in the United States, 49.4% of 18- to 34-year-olds are male, do you consider this a good representative sample? Why?

9. From Anderson et al. (2020), Chapter 4, #33.

Students taking the Graduate Management Admissions Test (GMAT) were asked about their undergraduate major and intent to pursue their MBA as a full-time or part-time student. A summary of their responses follows.

		Majors			Totals
		Business	Engineering	Other	
Intended Enrollment Status	Full-Time	352	197	251	800
	Part-Time	150	161	194	505
Totals		502	358	445	1305

- (a) Develop a joint probability table for these data.
- (b) Use the marginal probabilities of undergraduate major (business, engineering, or other) to comment on which undergraduate major produces the most potential MBA students.
- (c) If a student intends to attend classes full-time in pursuit of an MBA degree, what is the probability that the student was an undergraduate engineering major?
- (d) If a student was an undergraduate business major, what is the probability that the student intends to attend classes full-time in pursuit of an MBA degree?
- (e) Let A denote the event that the student intends to attend classes full-time in pursuit of an MBA degree, and let B denote the event that the student was an undergraduate business major. Are events A and B independent? Justify your answer.

10. From Anderson et al. (2020), Chapter 4, #37.

A 2018 Pew Research Center survey found that more Americans believe they could give up their televisions than could give up their cell phones (Pew Research website). Assume that the following table represents the joint probabilities of Americans who could give up their television or cell phone.

		Give Up Television		
		Yes	No	
Could Give Up Cellphone	Yes	.31	.17	.48
	No	.38	.14	.52
		.69	.31	

- (a) What is the probability that a person could give up her cell phone?
- (b) What is the probability that a person who could give up her cell phone could also give up television?
- (c) What is the probability that a person who could not give up her cell phone could give up television?
- (d) Is the probability a person could give up television higher if the person could not give up a cell phone or if the person could give up a cell phone?

11. From Anderson et al. (2020), Chapter 4, #38.

The Institute for Higher Education Policy, a Washington, D.C.-based research firm, studied the payback of student loans for 1.8 million college students who had student loans that began to become due six years ago (The Wall Street Journal). The study found that 50% of the student loans were being paid back in a satisfactory fashion, whereas 50% of the student loans were delinquent. The following joint probability table shows the probabilities of the student loan status and whether or not the student had received a college degree.

		College Degree		
		Yes	No	
Loan Status	Satisfactory	.26	.24	.50
	Delinquent	.16	.34	.50
		.42	.58	

- What is the probability that a student with a student loan had received a college degree?
- What is the probability that a student with a student loan had not received a college degree?
- Given the student had received a college degree, what is the probability that the student has a delinquent loan?
- Given the student had not received a college degree, what is the probability that the student has a delinquent loan?
- What is the impact of dropping out of college without a degree for students who have a student loan?

12. From Anderson et al. (2020), Chapter 4, #43.

According to a 2018 article in Esquire magazine, approximately 70% of males over age 70 will develop cancerous cells in their prostate. Prostate cancer is second only to skin cancer as the most common form of cancer for males in the United States. One of the most common tests for the detection of prostate cancer is the prostate-specific antigen (PSA) test. However, this test is known to have a high false-positive rate (tests that come back positive for cancer when no cancer is present). Suppose there is a .02 probability that a male patient has prostate cancer before testing. The probability of a false-positive test is .75, and the probability of a false-negative (no indication of cancer when cancer is actually present) is .20.

- What is the probability that the male patient has prostate cancer if the PSA test comes back positive?
- What is the probability that the male patient has prostate cancer if the PSA test comes back negative?
- For older men, the prior probability of having cancer increases. Suppose that the prior probability of the male patient is .3 rather than .02. What is the probability that the male patient has prostate cancer if the PSA test comes back positive? What is the probability that the male patient has prostate cancer if the PSA test comes back negative?
- What can you infer about the PSA test from the results of parts (a), (b), and (c)?

13. From Anderson et al. (2020), Chapter 4, #44.

ParFore created a website to market golf equipment and golf apparel. Management would like a special pop-up offer to appear for female website visitors and a different special pop-up offer to appear for male website visitors. From a sample of past website visitors, ParFore's management learned that 60% of the visitors are male and 40% are female.

- What is the probability that a current visitor to the website is female?
- Suppose 30% of ParFore's female visitors previously visited the Dillard's department store website and 10% of ParFore's male visitors previously visited the Dillard's department store website. If the current visitor to

ParFore's website previously visited the Dillard's website, what is the revised probability that the current visitor is female? Should the ParFore's website display the special offer that appeals to female visitors or the special offer that appeals to male visitors?

14. From Anderson et al. (2020), Chapter 4, #45.

The National Center for Health Statistics, housed within the U.S. Centers for Disease Control and Prevention (CDC), tracks the number of adults in the United States who have health insurance. According to this agency, the uninsured rates for Americans in 2018 are as follows: 5.1% of those under the age of 18, 12.4% of those ages 18-64, and 1.1% of those 65 and older do not have health insurance (CDC website). Approximately 22.8% of Americans are under age 18, and 61.4% of Americans are ages 18 – 64.

- (a) What is the probability that a randomly selected person in the United States is 65 or older?
- (b) Given that the person is an uninsured American, what is the probability that the person is 65 or older?

Remarks: Many problems are taken from Anderson et al. (2020). If possible you should do more problems from there.

References:

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., Cochran, J. J., Fry, M. J. and Ohlmann, J. W. (2020), *Statistics for Business & Economics*, 14th edn, Cengage, Boston, MA.