Problem Set - 2 (Math Recap - Sets, Functions and Counting Problems)

ECO 104 (Section 8)
Instructor: Shaikh Tanvir Hossain

Due Date : 16th November, 2023, Thursday (submit in class), individual assignment

§

You should write the solutions neatly in paper and submit in class. **This is NOT a group assignment, this is an individual assignment**, so please do the problems and submit individually! Please ask me in Ed if you have any question.

1. Suppose that $A \subset B$. Using venn diagram roughly explain why $B^c \subset A^c$.

> **Solution:** Draw the diagram, and you should see that A^c is bigegr than B^c.

2. In the following you have sets written with a set builder notation, write down all the elements of the sets using enumeration method, in other words write down all of the elements of the sets.
   (a) If the set C is defined as $C = \{x : x \in \mathbb{Z} \text{ and } -2 \leq x < 5\}$, then $C =?$
   (b) If the set D is defined as $D = \{x^2 : x \in \mathbb{N} \text{ and } 2 \leq x < 10\}$, then $D =?$

> **Solution:**
>
> (a) C = {-2, -1, 0, 1, 2, 3, 4}
>
> (b) D = {4, 9, 16, 25, 36, 49, 64, 81}

3. If the universal set is given by $S = \{1, 2, 3, 4, 5, 6\}$, and $A = \{1, 2\}$, $B = \{2, 4, 5\}, C = \{1, 5, 6\}$ are three sets, find the following sets:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) A^c
   (d) B^c
   (e) Check De Morgan's law by finding $(A \cup B)^c$ and $A^c \cap B^c$.
   (f) Check the distributive law by finding $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$.

> **Solution:**

4. Suppose we have following sets, which are intervals on the real line

$$A = \{x : 1 \leq x \leq 5\} = [1, 5],$$
$$B = \{x : 3 < x \leq 7\} = (3, 7],$$
$$C = \{x : x \leq 0\} = (-\infty, 0].$$

Here are the sets in the numberline

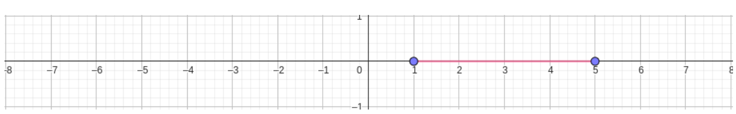

Figure 1: Set A

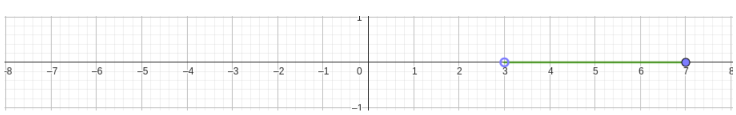

Figure 2: Set B

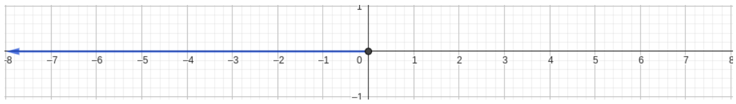

Figure 3: Set C

Now find out following intervals

(a) $A \cap B$

(b) $A \cap C$

(c) $A \cup B$

(d) $B \cup C$

(e) $A \cap B \cap C$

(f) C^c

(g) $B \cap C^c$

(h) $A \cup B \cup C$ (Hint: apply Associative Law)

(i) $A \cap (B \cup C)$ (Hint: apply Distributive Law)

(j) $A^c \cap B^c \cap C^c$ (Hint: apply DeMorgan's Law)

Some Hints:

To help you out, here I will do number a) and d).

a. $A \cap B = (3, 5]$

d. $B \cup C = (-\infty, 0] \cup (3, 7]$

To solve these problems it will be helpful if you draw the numberline

Solution:

(a) $A \cap B = (3, 5]$

(b) $A \cap C = \emptyset$

(c) $A \cup B = [1, 7]$

(d) $B \cup C = (-\infty, 0] \cup (3, 7]$

(e) $A \cap B \cap C = \emptyset$

(f) $C^c = (0, \infty)$

(g) $B \cap C^c = [0, 3)$

(h) $A \cup B \cup C = (-\infty, 0] \cup [1, 7]$

(i) $A \cap (B \cup C) = (3, 5]$

(j) $A^c \cap B^c \cap C^c = (0, 1) \cup (7, \infty)$.

5. An experiment has 3 parts. There are 3 possible outcomes for the first part, 2 for the second, and 4 for the third. How many ways we can perform the experiment?

Solution:

This is a simple application of multiplication rule, it should be $3 \times 2 \times 4$

6. Suppose we have 6 letters, $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, and F.

(a) How many ways we can select 3 letters from the group of 6 letters?

(b) How many ways we can order/arrange 3 letters from the group of 6 letters?

Solution:

(a) You should carefully notice the word "select". So for the first question, it's a combination problem. So the answer should be ${}^6C_3$.

(b) The second question specifically mentions "order", so this is a permutation problem, and the answer should be ${}^6P_3$

7. Suppose you throw 5 balls into 5 boxes, and one ball cannot be thrown to more than one boxed

(a) How many ways the 5 balls can be thrown into 5 boxes?

(b) If now we have 10 boxes, then how many ways the 5 balls can be thrown to 10 boxes?

Solution:

(a) 5 balls can be thrown to 5 boxes in $5 \times 4 \times 3 \times 2 \times 1$ possible ways (note that, this is same as 5! or ${}^5P_5$

(b) If we have 10 boxes then the answer should be $10 \times 9 \times 8 \times 7 \times 6$ possible ways. And again, recall, this is same as ${}^{10}P_5$.

8. Suppose in a school assembly some children need to be lined up

(a) If we have 5 children, how many ways they can be lined up?

(b) Now suppose that we have 10 children, 5 are to be chosen and lined up. How many ways they can be lined up, or in other words how many different lines are possible?

Solution:

(a) This is also an ordering problem. So the answer to the first question is $5!$,

(b) The answer to the second question is ${}^{10}P_5$.

9. Consider the experiment of tossing a coin 3 times.

(a) Develop a tree diagram for the experiment.

(b) List all the experimental outcomes.

Solution:
We have done this problem many times now, so you should be able to solve this problem.

10. If 4 dice are rolled and this is an experiment

(a) What is the total possible number of outcomes of the experiment? or How many ways all 4 dice can appear together?

(b) How many times 4 numbers will be different?

(c) How many times 4 numbers will be same?

Solution:

(a) if we roll 4 dice then there are $6^4 = 1296$ possible ways we can get the 4 dice together, or there are 1296 possible outcomes of the experiment.

(b) If we want 4 dices to be different than there are $6 \times 5 \times 4 \times 3 = 360$ possible ways we can get the 4 dice together.

(c) The answer is (a) - (b)

11. An elevator in a building starts with 5 passengers and can stop at any of the 7 floors. If every passenger has a possibility to get off at each floor and all the passengers leave independently of each other, then
    (a) What is the total number of options for each passenger.
    (b) What is the total number of options for all the passengers together? Or in other words, how many ways all passengers can leave?
    (c) If we know that all 5 passengers will leave to different floors then how many ways all passengers can leave?

Solution:

(a) Each passanger has 7 possible floors, so each passenger has 7 options.

(b) The total number of options for all the passengers together is $7^5 = 16807$, or there are possible ways 16807 they can leave.

(c) If we want them to leave in different floors, then the first passanger has 7 possible ways, the second one has 6, third one has 5... and so on... in this way we get $7 \times 6 \times 5 \times 4 \times 3 = 2520$ possible ways.